**Math 231 – HW 7.5 Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

***Remember -- FORMAT is as important as CONTENT – get them both right!***

3.3 11 3.6 10, 23, 6.

***Warm-up Questions -- Negations of Quantified Statements.***

|  |  |
| --- | --- |
| *The original quantified statement, in words:*  Any rational number is also a real number. | *The negation of the original statement, in words:* |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

|  |  |
| --- | --- |
| *The original quantified statement, in words:*  There is a rational number that is also an integer. | *The negation of the original statement, in words:* |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

**3.3 (11)** Do a formal proof of the theorem:

Theorem: If n = 4k+1 (for some integer k), then n2-1 is divisible by 8.

Proof:

**3.3 (29)** Use the sieve of Eratosthenes to find all the prime numbers less than 100. (read the rest of the problem in the book!)



Here's the list of prime numbers less than 100:

**3.6 (10)** Prove the theorem in two ways -- by contraposition and by contradiction.

Theorem: If the square of an integer is odd, then the original integer is odd.

*By contraposition:*

Exploration: Rewrite the theorem as an if-then conditional:

∀ n∈, if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Write the contraposition of your if-then statement:

∀ n∈, if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*Now, let's do the proof by contraposition:*

Theorem: If the square of an integer is odd, then the original integer is odd.

Proof: It is sufficient to show that: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(insert your contrapositive statement).

*By contradiction:*

Exploration: Write the negation of the theorem:

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*Now, let's do the proof by contradiction:*

Theorem: If the square of an integer is odd, then the original integer is odd.

Proof: Suppose not. In other words \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(insert your negation of the theorem).

**3.6 (23)** Prove the theorem in two ways -- by contraposition and by contradiction.

Theorem: If r is any rational number, and s is any irrational number, then  is irrational.

*By contraposition:*

Exploration: Rewrite the theorem as an if-then conditional:

∀  , if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Write the contraposition of your if-then statement:

∀  , if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, then \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*Now, let's do the proof by contraposition:*

Theorem: If r is any rational number, and s is any irrational number, then  is irrational.

Proof: It is sufficient to show that: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(insert your contrapositive statement).

*By contradiction:*

Exploration: Write the negation of the theorem:

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*Now, let's do the proof by contradiction:*

Theorem: If r is any rational number, and s is any irrational number, then  is irrational.

Proof: Suppose not. In other words \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(insert your negation of the theorem).

**3.6 (6)** Finish the following proof.

Theorem: The difference of any rational and any irrational number is irrational.

Proof: Suppose not. In other words, suppose that there is a rational number x and an irrational number y such that  is rational.

*Extra question: Rewrite the above statement using symbols instead of words.*

Since x is rational,